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THE RAY METHOD OF SOLVING THE THREE-DIMENSIONAL DYNAMIC PROBLEM OF COUPLED THERMOELASTICITY FOR A SPHERE†

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The three-dimensional problem of coupled thermoelasticity for a sphere is considered taking into account the finite rate of heat propagation. Solutions for the temperature and stress in a sphere heated by two dome-shaped energy fluxes are found by the ray method. Graphs for the temperatures and radial stresses are presented. © 1999 Elsevier Science Ltd. All rights reserved.

Most of the solutions of dynamic problems of thermoelasticity relate to bodies of infinite or semi-infinite dimension [1, 2].

1. The dynamic problem of coupled thermoelasticity is described by the equations of motion [2], the generalized heat conduction law [3], the law of conservation of energy, Duhamel–Neumann relations and Cauchy's relations [2]. After eliminating the strain tensor and transforming the equations, the system can be written in the dimensionless form [4]

$$\sigma_{ij,j} = \upsilon_{i,t}, \quad \tau q_{i,t} + q_i = -T_{,i}, \quad q_{i,i} + \delta \upsilon_{k,k} + T_{,t} = 0$$
$$\sigma_{ij,t} = ((1-2\eta)\upsilon_{k,k} - T_{,t})\delta_{ij} + \eta(\upsilon_{i,j} + \upsilon_{j,i}) \tag{1.1}$$

where σ_{ij} are the components of the stress tensor, υ_i are the components of the vector of displacement rates, q_i are the components of the heat flux vector, T is the temperature, τ is the relaxation time of the heat flux, δ is the thermomechanical coupling coefficient, t is the time, $\eta = \mu/(\lambda + 2\mu)$, λ and μ are the Lamé parameters and δ_{ij} is the Kronecker delta.

Suppose that there are no stresses on the boundary of a sphere of radius r_0 and that two dome-shaped energy fluxes operate.

The boundary conditions are

$$q_i(\theta, \varphi, r_0, t)\nu_i = g_0(t)\exp(-dr_0^2 \sin^2\theta)$$
$$\sigma_{ij}(\theta, \varphi, r_0, t)\nu_j = 0 \tag{1.2}$$

Here r, θ, φ are spherical coordinates, d is a constant and ν_i are the components of the unit normal to the surface.

2. The solution for the stresses, temperature, displacement rates and heat fluxes will be sought in the form of the radial series

$$f = (f^+ - [f])|_\Sigma - h(f_{,n}^{(1)+} - [f_{,n}^{(1)}])|_\Sigma + \frac{h^2}{2!}(f_{,nn}^{(2)+} - [f_{,nn}^{(2)}])|_\Sigma - \ldots \tag{2.1}$$
$$f_{,n\ldots n}^{(k)} = \frac{\partial^k f}{\partial x_i \partial x_j \ldots \partial x_l}\nu_i\nu_j\ldots\nu_l, \quad [f] = (f^+ - f^-)|_\Sigma$$

Here h is the distance along the normal to the front of the surface of strong discontinuity. The superscript plus denotes the value of the function ahead of the wave front of the discontinuity and the superscript minus denotes the value behind it.

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In a thermoelastic material we know [5] that two fronts of vortex-free waves (VFW) propagate with velocities

$$c_N = (1+\tau+\tau\delta \pm \sqrt{(1+\tau+\tau\delta)^2 - 4\tau})/(2\tau), \quad N=1,2$$

and one front of an equivoluminous wave (EW) propagates with velocity $c_3 = \sqrt{\eta}$. The equations for the zero terms of the series (there will be no summation over N and M below)

$$\begin{gathered} \delta\omega_N/\delta t + b_N\omega_N = c_N\Omega\omega_N, \quad \delta[v_i]/\delta t = c_3\Omega[v_i] \\ \omega_N = [v_i]\nu_i|_{\Sigma_N}, \quad b_N = \delta/(2[\tau\delta + (1-\tau c_N^2)^2]) \end{gathered} \tag{2.2}$$

Here $\delta/\delta t$ is the δ-derivative with respect to time [6].

On the VFW front the discontinuities of the physical parameters have the form

$$\begin{gathered} [q_i] = -\delta\omega_N\nu_i/(1-\tau c_N^2), \quad [T] = -\delta\tau c_N\omega_N/(1-\tau c_N^2) \\ -c_N[\sigma_{ij}] = (c_N^2 - 2\eta)\omega_N\delta_{ij} + 2\eta\omega_N\nu_i\nu_j, \quad [v_i] = \omega_N\nu_i \end{gathered} \tag{2.3}$$

On the EW front

$$\begin{gathered} [q_i] = 0, \quad [T] = 0, \quad [v_i]\nu_i = 0 \\ -c_3[\sigma_{ij}] = \eta([v_i]\nu_j + [v_j]\nu_i) \end{gathered} \tag{2.4}$$

For boundary-value problem (1.1), (1.2), the surfaces of discontinuity $\Sigma(t)$ are spherical waves which propagate towards the centre of the sphere. The curvilinear coordinates on the moving surface are taken as $y_1 = \theta, y_2 = \varphi$ $(0 \leqslant \theta \leqslant \pi, 0 \leqslant \varphi \leqslant 2\pi)$. The Cartesian coordinates x_i of the moving surface and the components of the unit normal to the surface ν_i have the form

$$\begin{gathered} x_1 = (r_0 - ct)\sin\theta\cos\varphi, \quad x_2 = (r_0 - ct)\sin\theta\sin\varphi, \quad x_3 = (r_0 - ct)\cos\varphi \\ \nu_1 = -\sin\theta\cos\varphi, \quad \nu_2 = -\sin\theta\sin\varphi, \quad \nu_3 = -\cos\varphi \end{gathered}$$

The components of the fundamental contravariant metric tensor $g^{\alpha\beta}$ on the surface, the coefficients of the second and third basic quadratic forms $b_{\alpha\beta}$, $c_{\alpha\beta}$, the average and Gaussian curvatures Ω and K have the form [7]

$$g^{\alpha\beta} = \begin{Vmatrix} K & 0 \\ 0 & K\sin^{-2}\theta \end{Vmatrix}, \quad b_{\alpha\beta} = \begin{Vmatrix} \Omega^{-1} & 0 \\ 0 & \Omega^{-1}\sin^{-2}\theta \end{Vmatrix}$$

$$c_{\alpha\beta} = \begin{Vmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{Vmatrix}, \quad \Omega = (r_0 - ct)^{-1}, \quad K = (r_0 - ct)^{-2}$$

For spherical waves, solutions (2.2) have the form

$$\begin{gathered} \omega_N = \omega_{0N}(y_1, y_2)r_0\exp(-b_N t)/(r_0 - c_N t) \\ [v_i] = v_{i0}(y_1, y_2)r_0/(r_0 - c_N t) \end{gathered} \tag{2.5}$$

The radial series (2.1) can be represented in the form

$$\begin{gathered} f(r,\theta,\varphi,t) = -\sum_{k=0}^{\infty}\frac{(r-r_0-c_1t)^k}{k!}[f^{(k)}_{,n\ldots n}]|_{\Sigma_1} - \\ -\sum_{k=0}^{\infty}\frac{(r-r_0-c_2t)^k}{k!}[f^{(k)}_{,n\ldots n}]|_{\Sigma_2} - \sum_{k=0}^{\infty}\frac{(r-r_0-c_3t)^k}{k!}[f^{(k)}_{,n\ldots n}]|_{\Sigma_3} \end{gathered} \tag{2.6}$$

The partial derivatives of the function f are related to the derivatives in curvilinear coordinates by kinematic and geometric consistency conditions of the $(m + 1)$th order [6, 8]

$$[f^{(m+1)}_{,in\ldots n}] = [f^{(m+1)}_{,n\ldots n}]\nu_i + g^{\alpha\beta}[f^{(m)}_{,n\ldots n}]_{,\alpha} x_{i,\beta} + L^{(m-1)}_{\beta}[f]x_{i,\beta}$$

$$[f^{(m+1)}_{,tn\ldots n}] = -c[f^{(m+1)}_{,n\ldots n}] + \delta[f^{(m)}_{,n\ldots n}]/\delta t + mg^{\alpha\beta}c_{,\beta}[f^{(m-1)}_{,n\ldots n}]_{,\alpha} +$$
$$+\sum_{R=2}^{m} C^{m-R+1}_{m} R!\, g^{\alpha_1\beta}c_{,\beta}B^{\alpha_R}_{\alpha_1}[f^{(m-R)}_{,n\ldots n}]_{,\alpha_R} \tag{2.7}$$

Here

$$L^{(m-1)}_{\beta}[f] = \sum_{R=2}^{m+1} C^{m-R+1}_{m}(R-1)!\, g^{\alpha_1\beta}[f^{(m-R+1)}_{,n\ldots n}]_{,\alpha_R} B^{\alpha_R}_{\alpha_1}$$

$$B^{\alpha_R}_{\alpha_1} = \prod_{N=2}^{R} b_{\alpha_{N-1}\beta_N} g^{\alpha_N\beta_N}$$

We differentiate system (1.1) m times with respect to the normal n and record at discontinuities. Using the kinematic and geometric consistency conditions (2.7), eliminating discontinuities $[\sigma^{(m+1)}_{ij,n\ldots n}]$, $[q^{(m+1)}_{i,n\ldots n}]$, $[T^{(m+1)}_{,n\ldots n}]$ and taking into account that the velocities of VFW and EW take constant values, we obtain

$$(\delta\tau c^2 - (1-\eta)(1-\tau c^2))[\upsilon^{(m+1)}_{k,n\ldots n}]\nu_k\nu_i + (c^2-\eta)(1-\tau c^2)[\upsilon^{(m+1)}_{i,n\ldots n}] =$$
$$= -\tau c^2(g^{\alpha\beta}[q^{(m)}_{k,n\ldots n}]_{,\alpha}x_{k,\beta}\nu_i + L^{(m-1)}_{\beta}[q_k]x_{k,\beta}\nu_i) -$$
$$-\delta\tau c^2(g^{\alpha\beta}[\upsilon^{(m)}_{k,n\ldots n}]_{,\alpha}x_{k,\beta}\nu_i + L^{(m-1)}_{\beta}[\upsilon_k]x_{k,\beta}\nu_i) -$$
$$-\tau c^2\nu_i\delta[T^{(m)}_{,n\ldots n}]/\delta t - \tau c\nu_k\nu_i\delta[q^{(m)}_{k,n\ldots n}]/\delta t -$$
$$-c[q^{(m)}_{k,n\ldots n}]\nu_k\nu_i + (1-\tau c^2)(-c(g^{\alpha\beta}[\sigma^{(m)}_{ij,n\ldots n}]_{,\alpha}x_{j,\beta} +$$
$$+L^{(m-1)}_{\beta}[\sigma_{ij}]x_{j,\beta}) + c\delta[\upsilon^{(m)}_{i,n\ldots n}]/\delta t - \nu_j\delta[\sigma^{(m)}_{ij,n\ldots n}]/\delta t - \nu_i\delta[T^{(m)}_{,n\ldots n}]/\delta t +$$
$$+(1-2\eta)(g^{\alpha\beta}[\upsilon^{(m)}_{k,n\ldots n}]_{,\alpha}x_{k,\beta}\nu_i + L^{(m-1)}_{\beta}[\upsilon_k]x_{k,\beta}\nu_i) +$$
$$+\eta(g^{\alpha\beta}[\upsilon^{(m)}_{k,n\ldots n}]_{,\alpha}x_{i,\beta}\nu_k + L^{(m-1)}_{\beta}[\upsilon_k]x_{i,\beta}\nu_k)) \tag{2.8}$$

Putting $c = c_3$ in the tensor equation (2.8), multiplying by ν_i and summing over i we obtain

$$(\delta\tau c_3^2 - (1-\eta)(1-\tau c_3^2))[\upsilon^{(m+1)}_{k,n\ldots n}]\nu_k =$$
$$= -\tau c_3^2(g^{\alpha\beta}[q^{(m)}_{k,n\ldots n}]_{,\alpha}x_{k,\beta} + L^{(m-1)}_{\beta}[q_k]x_{k,\beta}) -$$
$$-\delta\tau c_3^2(g^{\alpha\beta}[\upsilon^{(m)}_{k,n\ldots n}]_{,\alpha}x_{k,\beta} + L^{(m-1)}_{\beta}[\upsilon_k]x_{k,\beta}) -$$
$$-\tau c_3^2\delta[T^{(m)}_{,n\ldots n}]/\delta t - \tau c_3\nu_k\delta[q^{(m)}_{k,n\ldots n}]/\delta t - c_3[q^{(m)}_{k,n\ldots n}]\nu_k +$$
$$+(1-\tau c_3^2)(-c_3(g^{\alpha\beta}[\sigma^{(m)}_{ij,n\ldots n}]_{,\alpha}x_{j,\beta} +$$
$$+L^{(m-1)}_{\beta}[\sigma_{ij}]x_{j,\beta}\nu_i) + c_3\nu_i\delta[\upsilon^{(m)}_{i,n\ldots n}]/\delta t -$$
$$-\nu_j\nu_i\delta[\sigma^{(m)}_{ij,n\ldots n}]/\delta t - \delta[T^{(m)}_{,n\ldots n}]/\delta t +$$
$$+(1-2\eta)(g^{\alpha\beta}[\upsilon^{(m)}_{k,n\ldots n}]_{,\alpha}x_{k,\beta} + L^{(m-1)}_{\beta}[\upsilon_k]x_{k,\beta})) \tag{2.9}$$

Multiplying Eq. (2.8) by ν_i, summing over i and putting $c = c_N$, we obtain a differential equation for the change of $\omega^{(m)}_{nN} = [\upsilon^{(m)}_{i,n\ldots n}]\nu_i\,|\Sigma_N$ on the VFW front

$$\delta\omega^{(m)}_{nN}/\delta t + b_N\omega^{(m)}_{nN} - c_N\Omega\omega^{(m)}_{nN} = W^{(m-1)}(\omega_N) \tag{2.10}$$

Here $W^{(m-1)}(\omega_N)$ is a function of the derivatives along the normal ω_N of order $m-1$ and below.

Substituting (2.9) into system (2.8) with $c = c_3$, we obtain a system of differential equations for the components of the vector $[\upsilon^{(m)}_{i,n...n}]$ on the EW front

$$\delta[\upsilon^{(m)}_{i,n...n}]/\delta t - c_3\Omega[\upsilon^{(m)}_{i,n...n}] = W_3^{(m-1)} \tag{2.11}$$

The expression on the right-hand side does not contain $[\upsilon^{(m)}_{i,n...n}]$.

If $m = 1$, we obtain a first-order differential equation from (2.10) and (2.11) from which to find $\omega^{(1)}_{nN} = [\upsilon^{(1)}_{i,n}]\nu_i$ and $[\upsilon^{(1)}_{i,n}]$

$$\begin{gathered}\delta\omega^{(1)}_{nN}/\delta t + (b_N - c_N/(r_0 - c_N t))\omega^{(1)}_{nN} = \\ = a_N\omega_{0N}r_0\exp(-b_N t)/(r_0 - c_N t) - \\ -F_T c_{3-N} r_0 \exp(-b_N t)/(r_0 - c_N t)^3 \\ \delta[\upsilon^{(1)}_{i,n}]/\delta t = c_3[\upsilon^{(1)}_{i,n}]/(r_0 - c_N t)\end{gathered} \tag{2.12}$$

Here

$$F_T = dr_0^2(1 + 2dr_0^2\sin^2\theta\cos^2\theta - 2\cos^2\theta) - 2$$

On the VFW front we have

$$\begin{gathered}[T^{(1)}_{,n}] = \tau(1-\tau c_N^2)^{-2}\{-\tau c_N(1-\tau c_N^2)\omega^{(1)}_{nN} + \\ +(1 - \tau b_N(1+\tau c_N^2))\omega_N + \tau c_N(1-\tau c_N^2)\Omega\omega_N\} \\ [q^{(1)}_{i,n}] = \delta(1-\tau c_N^2)^{-2}\{-(1-\tau c_N^2)\omega^{(1)}_{nN}\nu_i + \\ +c_N(1-2\tau b_N)\omega_N\nu_i - (1-\tau c_N^2)g^{\alpha\beta}\omega_{N,\alpha}x_{i,\beta}\} \\ -c_N[\sigma^{(1)}_{ij,n}] = \{(c_N^2 - 2\mu)\omega^{(1)}_{nN} + b_N c_N^{-1}(c_N^2 + 2\mu)\omega_N - \\ -(c_N^2 - 2\mu)\Omega\omega_N\}\delta_{ij} + 2\mu(\omega^{(1)}_{nN} - b_N c_N^{-1}\omega_N + \Omega\omega_N)\nu_i\nu_j \\ -\mu g^{\alpha\beta}g^{\sigma\gamma}b_{\alpha\sigma}\omega_N(x_{i,\beta}x_{j,\gamma} + x_{j,\beta}x_{i,\gamma}) + 2\mu g^{\alpha\beta}\omega_{N,\alpha}(x_{i,\beta}\nu_j + x_{j,\beta}\nu_i) \\ [\upsilon^{(1)}_{i,n}] = \omega^{(1)}_{nN}\nu_i + g^{\alpha\beta}\omega_{N,\alpha}x_{i,\beta}\end{gathered} \tag{2.13}$$

On the EW front

$$\begin{gathered}[T^{(1)}_{,n}] = 0,\ [q^{(1)}_{i,n}] = 0,\ [\upsilon^{(1)}_{k,n}]\nu_k = -g^{\alpha\beta}[\upsilon_k]_{,\alpha}x_{k,\beta} \\ -c_3[\sigma^{(1)}_{ij,n}] = \mu([\upsilon^{(1)}_{i,n}]\nu_j + [\upsilon^{(1)}_{j,n}]\nu_i) + \mu([\upsilon_i]\nu_j + [\upsilon_j]\nu_i)\Omega + \\ +\mu g^{\alpha\beta}([\upsilon_i]_{,\alpha}x_{j,\beta} + [\upsilon_j]_{,\alpha}x_{i,\beta})\end{gathered} \tag{2.14}$$

The solution of the differential equations of system (2.12) has the form

$$\begin{gathered}\omega^{(1)}_{nN} = [\omega^{(1)}_{n0N}r_0/(r_0 - c_N t) + (a_N\omega_{0N}r_0/(r_0 - c_N t) - F_T r_0/(r_0 - c_N t)^2)]\exp(-b_N t) \\ [\upsilon^{(1)}_{i,n}] = \upsilon_{in0}r_0/(r_0 - c_3 t)\end{gathered} \tag{2.15}$$

3. Relations (2.3)–(2.6) and (2.13)–(2.15) can be used to construct the stress tensor, temperature, heat flux vector and displacement rates. Using (2.3)–(2.5), we substitute ray series (2.6) for the heat flux and stress tensor into (1.2) and let $t = 0$, $r = r_O$. Adding the condition on the EW $[\upsilon_k]\nu_k = 0$, we obtain a system of five equations

$$\delta\omega_{01}/(1-\tau c_1^2) + \delta\omega_{02}/(1-\tau c_2^2) = g_0(0)\exp(-dr_0^2\sin^2\theta) \tag{3.1}$$

$$(c_1\omega_{01} + c_2\omega_{02})\nu_i + c_3\upsilon_{i0} = 0,\ \upsilon_{i0}\nu_i = 0 \tag{3.2}$$

Note that condition (3.2) holds on any sufficiently smooth surface which is load-free. It follows from the solution of (3.2) that $\upsilon_{i0} = 0$. Thus ($[\upsilon_i] = 0$) we have shown that an EW is an acceleration wave in the case of a free boundary ($\sigma_{ij}\nu_j = 0$). It follows from the solution of system (3.1), (3.2) that

$$w_{0N} = F_n \exp(-dr_0^2 \sin^2 \theta) \tag{3.3}$$
$$F_N = c_M(1-\tau c_N^2)(1-\tau c_M^2)g_0(0)/(\delta(c_M(1-\tau c_M^2)-c_N(1-\tau c_N^2)))$$
$$M = 3-N$$

Using relations (2.3)–(2.5) and (2.13)–(2.15) we substitute the radial series (2.6) for the heat flux and stress tensor into boundary conditions (1.2). We differentiate the resulting equations with respect to time t and put $r = 0$, $r = r_0$. Adding the condition on the EW $[\upsilon_{k,n}^{(1)}] = -g^{\alpha\beta}[\upsilon_k]_{,\alpha}x_{k,\beta}$, we obtain a system of five equations for $\omega_{n0N}^{(1)}$, $\upsilon_{in0}^{(1)}$. The solution of this system is

$$\omega_{n0N}^{(1)} = F_{nN}^{(1)} \exp(-dr_0 \sin^2 \theta)$$
$$\upsilon_{inN}^{(1)} = \frac{1}{c_3^2}(S_0\nu_i + \sum_{N=1}^{2}(-c_N^2 F_{nN}^{(1)}\nu_i + 2c_3^2 r_0 F_N d \sin 2\theta \cdot x_{i,1})|_{\Sigma_N(0)})$$

Here

$$F_{nN}^{(1)} = (Q_0 c_M(1-\tau c_M^2) - S_0\delta)(1-\tau c_N^2)\times$$
$$\times(\delta c_N(c_M(1-\tau c_M^2)-c_N(1-\tau c_N^2)))^{-1}$$

$$Q_0 = \sum_{N=1}^{2}\left[\frac{c_N}{r_0}F_T + \frac{c_N}{r_0} - b_N + \frac{c_N^2(1-2\tau b_N)}{1-\tau c_N^2}\right]\times$$
$$\times\frac{c_M(1-\tau c_M^2)g_0(0)}{c_M(1-\tau c_M^2)-c_N(1-\tau c_N^2)}, \quad M = 3-N$$

$$S_0 = \sum_{N=1}^{2}\left[2\left(\frac{c_N^2}{r_0} - c_N b_N - \frac{2c_3^2}{r_0}\right) + \frac{c_N^2}{2}F_T\right]F_N$$

Differentiating boundary conditions (1.2) m times with respect to time t and putting $t = 0$, $r = r_0$, we can determine the subsequent coefficients of the series $\omega_{n0N}^{(m)}$, $\upsilon_{in0}^{(m)}$.

The solutions are illustrated in Fig. 1, which shows graphs of the variation of the radial stresses σ_{rr} and the temperature T against the time t for fixed depths $r = 0.95$ (the dashed curve) and $r = 0.85$ (the solid curve). The chosen material was aluminium $\delta = 0.028$, $\tau = 4.18$, $\eta = 1.94$, the unit of dimensionless time is 2.39 Ps, the unit of dimensionless temperature is 300°C and the unit of dimensionless stress is 1.41×10^9 Pa) the sphere radius $r_0 = 1$, $g_0(t) = 1$. The radial stresses and

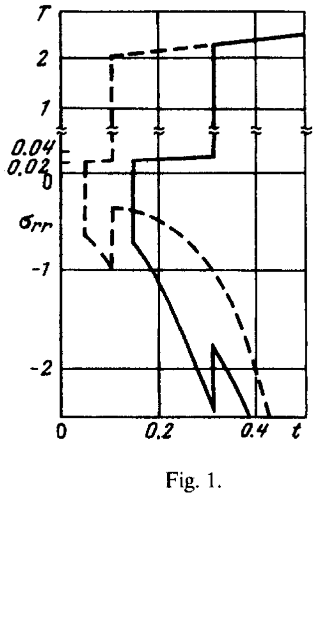

Fig. 1.

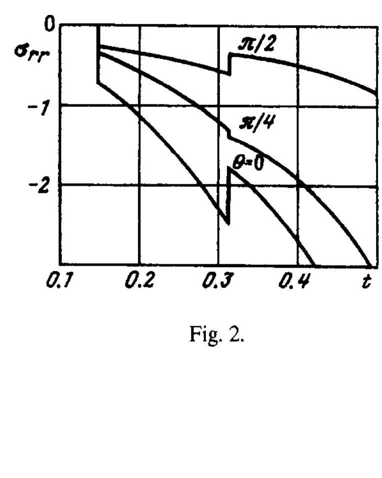

Fig. 2.

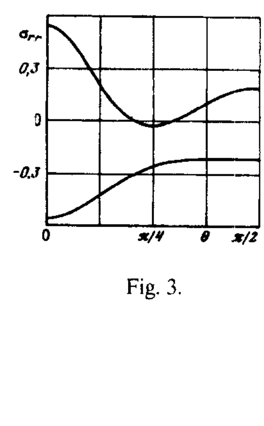

Fig. 3.

temperature experience a jump when the first front of the thermoelastic wave arrives. The compressive stresses and temperature increase until the second front arrives. This is because of the importance of the geometric factor in the problem for a sphere. Reducing the area of the surface which moves towards the sphere centre has a greater effect than unloading the material, as observed in the problem for a half-space [3]. The second front makes a major contribution to the change in temperature. The first front of the thermoelastic wave has an insignificant effect on the change in temperature and can be neglected, since the jump of the first temperature front is no more than 2% of that of the second.

Figure 2 shows the dependence of the radial stresses σ_{rr} on time t at $r = 0.85$ for values of the angle $\theta = 0$, $\theta = \pi/4$, $\theta = \pi/2$. It follows from the first boundary condition (1.2) that the heat flux on the boundary of the sphere decreases as θ increases from zero to $\pi/2$. The graphs show that the largest jump of the leading front of radial stress ($t = 0.147$) is observed when $\theta = 0$, and the smallest when $\theta = \pi/2$. Subsequently the compressive stresses increase with time until the second front arrives ($t = 0.312$). The second front causes a sudden decrease in the compressive radial stresses for $\theta = 0$ and $\theta = \pi/2$ (unloading of the material) and an increase for $\theta = \pi/4$ (loading of the material). The calculations show that the EW front arrives when $t = 0.298$ and has only a slight influence on the change of radial stress.

Figure 3 shows graphs of the discontinuities of the VFW of radial stresses σ_{rr} against the angle θ with $r = 0.85$. The leading front of the VFW (the lower curve), which moves with velocity $c_1 = 1.018$, in a monotone function. The thermal load which operates on the sphere boundary $r_0 = 1$ is a decisive factor in the formation of this front. The heat flux, which decreases as θ increases, gives rise to a smaller compressive radial stress σ_{rr}. The jump of the second VFW front (the upper curve), moving with velocity $c_2 = 0.480$, decreases in the segment $[0, \pi/4]$ and increases in the segment $[\pi/4, \pi/2]$. Positive values of the radial stress σ_{rr} result in unloading and negative values result in loading of the material. Loading of the material is clearly observed near the angle $\pi/4$.

These solutions apply to a thermoelastic body and can be used when designing the spherical mirrors of resonators of lasers subjected to the action of brief high-intensity heat fluxes.

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